Measuring the size of the Earth

Introduction

This document describes the method for an experiment to measure the circumference of the Earth. It relies on the sun angle elevation difference, measured at two different locations at the same time. The method was first used Eratosthesnes, who in 240 BCE calculated the circumference of the Earth using the sun elevation angles in Alexandria and Synene in Egypt. This experiment will be done between primary school students in Cairns and Melbourne, Australia.

One benefit of this experiment is that students will not have to simply accept the fact that the Earth is round and not flat, they will have measured it themselves.

Equipment

- One meter ruler or stick
- String
- Protractor
- (Optional) spirit level or plumb bob to ensure ruler is held vertically

Method

At 12:00 PM noon on the same day, in two different locations (Cairns and Melbourne), students will measure the sun elevation angle above the horizon. This will be done using a ruler held vertically and string (see figure below). The technical term for this device is a gnomon.
Calculation

The essential idea is that sun elevation angle difference will be approximately 21 degrees, i.e. approximately $\frac{1}{17}$ of a circle, and therefore the Earth circumference is 17 times the distance between the two cities.

The circumference $C$ of the Earth is

$$C = s \times \frac{360^\circ}{\theta}$$

Where $s$ is the distance between two places and $\theta$ is the sun elevation angle difference.

Explicitly, at 21 June 2018 at noon, the sun elevation will be:

- Cairns: 50°
- Melbourne: 29°

Students should be able to measure these angles to within a few degrees, so they will likely measure the angle difference to be 21 +/- 2 degrees or 19–23 degrees. The distance Melbourne to Cairns via straight line is approximately $s \approx 2320$ km.

Therefore, for the lower estimate using 23 degrees:

$$C \approx 2320 \times \frac{360^\circ}{23^\circ} \approx 36,000 \text{ km}$$

Therefore, for the upper estimate using 19 degrees:

$$C \approx 2320 \times \frac{360^\circ}{19^\circ} \approx 44,000 \text{ km}$$

So students should measure the earth within a range, to be 36,000 –44,000 km. Compare this to the accepted value of 40,007 km for the polar circumference (NASA)^3.

Uncertainty

Students should be encouraged to consider the uncertainty of the measurement, and how it could be improved. What happens if there is error in the distance measurement?

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Distance Cairns to Melbourne

- Straight line (great circle distance): 2316 km
- Along roads via Google maps: 2705 km

Contact

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5 https://goo.gl/maps/D2cp8NpxLEL2
Further material

From the US National Oceanic and Atmospheric Administration⁶:

⁶https://oceanservice.noaa.gov/education/tutorial_geodesy/media/supp_geo02a.html